$$\int \underbrace{xe^x \, dx}$$

$$= xe^x - e^x + C$$

diff:

$$\underbrace{xe^x + e^x} - \cancel{e^x} + C$$

differentiating xe^x :

(product rule)

$$(1)e^x + x(e^x) =$$

$$\frac{d}{dx}(xe^x) = xe^x + e^x$$

$$\int xe^x dx + \int e^x dx = xe^x$$

$$\int xe^x dx = xe^x - \int e^x dx$$

$$\int f(x) g(x) = f(x) G(x) - \int f'(x) G(x)$$

$$\int (f \cdot g)' = \int f' \cdot g + \int f \cdot g'$$

$$\Longrightarrow \quad f \cdot g = \int f' g + \int f g'$$

$$f \cdot g - \int f' g = \int f \cdot g'$$

Int by parts

$$uv - \int v\,du = \int u\,dv$$

$$\int u\,dv = uv - \int v\,du$$

$$\int x \ln x \, dx$$

Let $u = \ln x \qquad dv = \int x\,dx$

take deriv $\qquad du = \frac{1}{x}dx \qquad \boxed{v = \frac{x^2}{2}}$ antiderivative

$$\int x \ln x\,dx = (\ln x)\left(\frac{x^2}{2}\right) - \int \left(\frac{x^2}{2}\right)\left(\frac{1}{x}dx\right)$$

$$\int x \ln x\,dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx$$

$$\int x \ln x\,dx = \boxed{\frac{x^2}{2}\ln x - \frac{1}{2}\left(\frac{x^2}{2}\right) + C}$$

$$\int u\,dv = uv - \int v\,du$$

$$\int \ln x \, dx$$

Let $u = \boxed{\ln x}$ $dv = \boxed{1 \, dx}$

$du = \frac{1}{x} dx$ $v = x$

$$\int \ln x \, dx = x \ln x - \int (x) \frac{1}{x} dx$$

$$= x \ln x - \int 1 \, dx$$

$$= x \ln x - x + C$$

$$\int u\,dv = uv - \int v\,du$$

$$\int x \sin x\,dx =$$

Let $u = x$ $\qquad dv = \sin x \; dx$

$du = dx \qquad v = -\cos x$

$$uv - \int v\,du = x(-\cos(x)) - \int -\cos x\,dx$$

$$= x(-\cos(x)) - (-\sin(x))$$

$$\boxed{= x(-\cos(x)) + \sin(x) + C}$$

$$\int u\,dv = uv - \int v\,du$$

$$\int e^x \sin x\,dx$$

$$u = \sin x \qquad dv = e^x\,dx$$

$$du = \cos x\,dx \qquad v = e^x$$

$$\int e^x \sin x\,dx = e^x \sin x - \int e^x \cos x\,dx$$

$$\int e^x \cos x\,dx$$

$$u = \cos x \qquad dv = e^x\,dx$$

$$du = -\sin x\,dx \quad v = e^x$$

$$\int e^x \sin x\,dx = e^x \sin x - \left(e^x \cos x - \int e^x(-\sin x)\,dx\right)$$

$$\int e^x \sin x\,dx = e^x \sin x - e^x \cos x - \int e^x \sin x\,dx$$

$$+\int e^x \sin x\,dx \qquad\qquad +\int e^x \sin x\,dx$$

$$2\int e^x \sin x\,dx = e^x \sin x - e^x \cos x$$

$$\int xe^x\,dx =$$

$$xe^x - e^x + C$$

differentiate xe^x

$$(1)e^x + x(e^x)$$

$$\int \frac{d}{dx}(xe^x) = \int e^x + \int xe^x$$

$$xe^x = \int e^x dx + \int xe^x dx$$

$$xe^x - \int e^x dx = \int xe^x dx$$

☆

Integration by parts

$$\int (f \cdot g)' = \int f' g + \int f g'$$

$$f \cdot g = \int f' g + \int f g'$$

$$\int u\,dv = uv - \int v\,du$$

$$\underbrace{\int u\,dv} = uv - \int v\,du$$

$$\int x \ln x \, dx =$$

Let $\boxed{u = \ln x}$ $\boxed{dv = x \, dx}$

take deriv. $\boxed{du = \frac{1}{x}\,dx \;|\; v = \frac{x^2}{2}}$ take antideriv.

$$\int x \ln x \, dx = \left(\frac{x^2}{2}\right)(\ln x) - \int \left(\frac{x^2}{2}\right)\left(\frac{1}{x}\right) dx$$

$$= \frac{x^2}{2}\ln x - \frac{1}{2}\int x \, dx$$

$$\int x \ln x \, dx = \frac{x^2}{2}\ln x - \frac{1}{2}\left(\frac{x^2}{2}\right) + C$$

$$\int u\,dv = uv - \int v\,du$$

$$\int \ln x\,dx$$

$u = \ln x$

$du = \frac{1}{x}dx$

$dv = 1\,dx$

$v = x$

$$x(\ln x) - \int x\left(\frac{1}{x}\right)dx = x\ln x - x + C$$

$$\int u\,dv = uv - \int v\,du$$

$$\int x\sin x\,dx$$

let $u = x$ $\qquad dv = \sin x\,dx$

$du = dx$ $\qquad V = -\cos x$

$$x(-\cos x) - \int(-\cos(x))\,dx$$

$$-x\cos x + \sin x + C$$

$U = \sin x$ $\qquad dU = x\,dx$

$du = \cos x\,dx$ $\qquad V = \frac{x^2}{2}$

$UV - \int v\,du$ $\qquad \frac{x^2\sin x}{2} - \int \frac{x^2\cos x}{2}\,dx$

$$\int u\,dv = uv - \int v\,du$$

$$\int e^x \sin x\,dx$$

Let $u = e^x$ $\quad dv = \sin x\,dx$

$du = e^x\,dx$ $\quad v = -\cos x$

$$\int e^x \sin x\,dx = -e^x \cos x - \int (-\cos x) e^x\,dx$$

$$\int e^x \sin x\,dx = -e^x \cos x + \int e^x \cos x\,dx \quad ☆$$

Let $u = e^x$ $\quad dv = \cos x\,dx$

$du = e^x\,dx$ $\quad v = \sin x$

$$\int e^x \sin x\,dx = -e^x \cos x + \left(e^x \sin x - \int e^x \sin x\,dx \right)$$

$$\int e^x \sin x\,dx = -e^x \cos x + e^x \sin x - \int e^x \sin x\,dx$$

$$+\int e^x \sin x\,dx \qquad\qquad + \int e^x \sin x\,dx$$

$$2\int e^x \sin x\,dx = -e^x \cos x + e^x \sin x$$

$$\int e^x \sin x\,dx = \frac{e^x \sin x - e^x \cos x}{2}$$

$$\int e^{3x}\,dx$$

$u = 3x$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$$\frac{1}{3}\int e^{u}\,du$$

$$= \frac{1}{3}e^{u} + C$$

$$= \frac{1}{3}e^{3x} + C$$

$$\frac{d}{dx}(b^{x}) =$$

$$(\ln b)b^{x}$$

$$\frac{d}{dx}(e^{x}) =$$

$$(\ln e)e^{x}$$